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Add- 4-E/10, Dabouli-I, Near Durga Mandir

CLASSES – V TO XII (CBSE, ICSE/ISC), IIT/NDA/TGT/PGT

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Class-12th TOPIC→ Determinants of Matrices

Q.1→ Evaluate the determinant $\Delta = \begin{vmatrix} \log_3 512 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix}$.

Q.2→ Prove that the determinant $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ is independent of θ.

Q.3→ If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$, then show that $|3A| = 27|A|$

Q.4→ Find the integral value of x, if $\begin{vmatrix} x^2 & x & 1 \\ 0 & 2 & 1 \\ 3 & 1 & 4 \end{vmatrix} = 28$.

Q.5→ Find the Equation of the line joining $A(1,3)$ and $B(0,0)$ using determinants and find k if $D(k,0)$ is a point such that area of ΔABD is 3 sq. units.

Q.6→ For the matrix $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, find the numbers a and b such that $A^2 + aA + bI = 0$ hence, find A^{-1}.

Q.7→ If matrix $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfies $A^T = A^{-1}$, find

Q.8→ If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$, find the value of λ so that $A^2 = \lambda A - 2I$. hence, find A^{-1}.

Q.9→ For the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$. Show that $A^3 - 6A^2 + 5A + 11I_3 = 0$ hence find A^{-1}.

Q.10→ Find the adjoint of the matrix $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ and hence show that $A(\text{adj}A) = |A| I_3$.

Q.11→ If $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, find A^{-1} and show that $A^{-1} = \frac{1}{2}(A^2 - 3I)$.

Q.12→ Solve the following system of equations by matrix method:

(i) $\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10$ (ii) $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$ (iii) $x - y + 2z = 7$

Q.13→ If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & 1 & 5 \end{bmatrix}$ are two square matrices, find AB and hence solve the system of linear equations:

$x - y = 3,\ 2x + 3y + 4z = 17,\ y + 2z = 7$

Class-12th TOPIC→ Determinants of matrices.

Q.14→ If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1} and hence solve the system of linear Equations:

$2x - 3y + 5z = 11$, $3x + 2y - 4z = -5$, $x + y - 2z = -3$

Q.15→ $A = \begin{bmatrix} 3 & -2 & 0 \\ 2 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}$, find A^{-1} and hence solve the following system of Equations:

$3x - 4y + 2z = -1$, $2x + 3y + 5z = 7$, $x + z = 2$

Q.16→ $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$, find AB. hence, solve the system of Equations.

$x - 2y = 10$, $2x + y + 3z = 8$ and $-2y + z = 7$.

Q.17→ Given $A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, find BA and use this to solve the Equations

$y + 2z = 7$, $x - y = 3$, $2x + 3y + 4z = 17$

Q.18→ If $A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 2 \\ -3 & 1 & -1 \end{bmatrix}$, find A^{-1} and hence solve the system of Equations

$2x + y - 3z = 13$, $3x + 2y + z = 4$, $x + 2y - z = 8$.

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CLASS- XII TOPIC- ALGEBRA OF MATICES

Q.1→ Construct a 3x2 matrix $A = [a_{ij}]$ whose Elements are given By

(i) $a_{ij} = e^{ix} \sin jx$ (ii) $a_{ij} = e^{-ix} \cos\left(\frac{\pi}{2} i + jx\right)$

Q.2→ Construct a 2x2 matrix $A = [a_{ij}]$ whose Elements a_{ij} are given by:

(i) $a_{ij} = \frac{(i-2j)^2}{2}$ (ii) $a_{ij} = \frac{|-3i+j|}{2}$

Q.3→ Construct a 3x4 matrix $A = [a_{ij}]$ whose Elements a_{ij} given By:

(i) $a_{ij} = \frac{1}{2}|-3i+j|$

Q.4→ If $\begin{bmatrix} xy & 4 \\ z+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix}$, then find the values of x, y, z and w.

Q.5→ Find a Matrix X such that $2A+B+X=0$, where $A = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$.

Q.6→ Find x, y, z and t, if

(i) $3\begin{bmatrix} x & y \\ z & t \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2t \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+t & 3 \end{bmatrix}$

(ii) $2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 14 \\ 15 & 14 \end{bmatrix}$

Q.7→ If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then find k so that $A^2 = 8A + kI$.

Q.9 → Let $A = \begin{bmatrix} 0 & -\tan(a/2) \\ \tan(a/2) & 0 \end{bmatrix}$ and I be the identity matrix of order 2. Show that

$$I + A = (I - A)\begin{bmatrix} \cos a & -\sin a \\ \sin a & \cos a \end{bmatrix}.$$

Q.10 → Let $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Show that $F(x)F(y) = F(x+y)$.

Q.11 → Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ and $f(x) = x^2 - 4x + 7$. Show that $f(A) = 0$. Use this result to find A^5.

Q.12 → If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I_2 = 0$.

Q.13 → Show that the matrix $A = \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$ is a root of the Equation $A^2 - 12A - I = 0$.

Q.14 → If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = 0$. use this to find A^4.

Q.15 → If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then find λ, μ so that $A^2 = \lambda A + \mu I$

Q.16 → If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, then show that A is a root of the polynomial $f(x) = x^3 - 6x^2 + 7x + 2$.

Q.17 → If $A = \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, show that $A^2 - 7A + 10I_3 = 0$.

Q.18 → If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, then find $A^2 - 5A - 14I$. Hence, obtain A^3.

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Q.19→ If A and B are symmetric matrices, then show that AB is symmetric iff AB = BA i.e. A and B commute.

Q.20→ Show that the matrix $B^T AB$ is symmetric or skew-symmetric according as A is symmetric or skew-symmetric.

Q.21→ Express the matrix $A = \begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 2 & 4 & 5 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

Q.22→ Express the matrix $A = \begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

Q.23→ Express the matrix $\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$ as the sum of a symmetric and skew-symmetric matrix and verify your result.

Q.24→ For the matrix $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$, find $A + A^T$ and verify that it is a symmetric matrix.

ANSWER SHEET

A.2 (i) $\begin{bmatrix} 1/2 & 9/2 \\ 0 & 2 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 1/2 \\ 5/2 & 2 \end{bmatrix}$

A.3 → (i) $\begin{bmatrix} 1 & 1/2 & 0 & 1/2 \\ 5/2 & 2 & 3/2 & 1 \\ 4 & 7/2 & 3 & 5/2 \end{bmatrix}$

A.4 → $x=2, y=4, z=-6, w=4$ or $x=4, y=2, z=-6, w=4$

A.5 → $\begin{bmatrix} -1 & -2 \\ -7 & -13 \end{bmatrix}$

A.6 → (i) $x=2, y=4, t=3, z=1$ (ii) $x=2, y=9$

A.7 → $K=-7$

A.14 → $\begin{bmatrix} 39 & 55 \\ -55 & -16 \end{bmatrix}$

A.15 → $\lambda=4, \mu=-1$

A.18 → $\begin{bmatrix} 187 & -195 \\ -156 & 148 \end{bmatrix}$

A.22 → Symmetric matrix = $\begin{bmatrix} 4 & 5/2 & 0 \\ 5/2 & 5 & 5/2 \\ 0 & 5/2 & 1 \end{bmatrix}$, Skew Symmetric matrix $\begin{bmatrix} 0 & -1/2 & -1 \\ 1/2 & 0 & 9/2 \\ 1 & -9/2 & 0 \end{bmatrix}$

A.23 → Symmetric matrix $\begin{bmatrix} 3 & 1/2 & -5/2 \\ 1/2 & -2 & -2 \\ -5/2 & -2 & 2 \end{bmatrix}$, Skew Symmetric matrix $\begin{bmatrix} 0 & -5/2 & -3/2 \\ 5/2 & 0 & -3 \\ 3/2 & 3 & 0 \end{bmatrix}$

A.24 → $A = \begin{bmatrix} 4 & 8 \\ 8 & 14 \end{bmatrix}$.