Q. 40. Diff. $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ w.r.t. $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$

Sol:- $u = \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$

Put $x = \sin\theta \Rightarrow \boxed{\theta = \sin^{-1}x}$

$u = \sec^{-1}\left(\frac{1}{\sqrt{1-\sin^2\theta}}\right)$

$u = \sec^{-1}\left(\frac{1}{\cos\theta}\right)$

$u = \sec^{-1}(\sec\theta)$

$u = \theta$

$u = \sin^{-1}x$

Diff w.r.t 'x'

$\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}}$ — (1)

$v = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

Put $x = \sin\theta \Rightarrow \theta = \sin^{-1}x$

$v = \sin^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right)$

$v = \sin^{-1}(2\sin\theta\cos\theta)$

$v = \sin^{-1}(\sin 2\theta)$

$v = 2\theta$

$v = 2\sin^{-1}x$

Diff - -

$\frac{dv}{dx} = \frac{2}{\sqrt{1-x^2}}$

$\Rightarrow \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{\frac{1}{\cancel{\sqrt{1-x^2}}}}{\frac{2}{\cancel{\sqrt{1-x^2}}}} = \frac{1}{2}$

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(46). if $x\sqrt{1+y} + y\sqrt{1+x} = 0$, then P.T. $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$

Sol:

$$\left(x\sqrt{1+y}\right)^2 = \left(-y\sqrt{1+x}\right)^2$$

$$x^2(1+y) = y^2(1+x)$$

$$x^2 + x^2y - y^2 - y^2x = 0$$

$$(x-y)(x+y) + xy(x-y) = 0$$

$$(x-y)\left[x+y+xy\right] = 0$$

$x = y$, $\quad x+y+xy = 0$

$$x+y(1+x) = 0$$

$$y(1+x) = -x$$

$$y = \frac{-x}{1+x}$$

$$y = -\frac{x}{1+x} \quad \text{--- (1)}$$

Diff - - -

$$\frac{dy}{dx} = -\left[\frac{(1+x)\frac{d}{dx}x - x\frac{d}{dx}(1+x)}{(1+x)^2}\right]$$

$$\frac{dy}{dx} = -\left[\frac{(1+x)\times 1 - x\times(0+1)}{(1+x)^2}\right]$$

$$\frac{dy}{dx} = -\left[\frac{1+x-x}{(1+x)^2}\right]$$

$$\boxed{\frac{dy}{dx} = -\frac{1}{(1+x)^2}}$$

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